लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2021-22

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Express $v=(x,\, y)$ as a linear combination of $v_1=(1,1)$ and $v_2=(1,-1)$ in $\mathbb{R}^2$.

(b) What 2 by 2 matrices represent the transformations that

(i) rotate every point by an angle θ about the origin.

(ii) reflect every point about the x-axis.

(c) What is the geometric object corresponding to the smallest subspace V_0 containing a nonzero vector $v=(r,\, s,\, t)\in\mathbb{R}^3$? Answer with reason.

(d) Write the matrix equation for the system of equations:

$$x+y=3,\, -3y+4z=17,\, x-z=-8\,.$$

(e) Is there any straight line in the vector space R_2 which is a subspace of R_2 ?

(f) Find the inverse of the matrix $A=\begin{bmatrix} 5 & 3 \\ -2 & 2 \end{bmatrix}$.

(g) For what values of z the three vectors (1, 1, 2), (z, 1, 1) and (1, 2, 1) are linearly independent?

(h) It is impossible for a system of linear equations to have exactly two solutions. Explain why.

(i) Prove that $\{(x,\, y,\, z)\in\mathbb{R}^3 : x^2+y^2=z^2\}$ is not a subspace of $\mathbb{R}^3$.

2. (a) Examine if the set S is a subspace of $\mathbf{R}_3$, $S=\{(x,\, y,\, z)\in \mathbf{R}_3 \mid x=0\ z=0\}$. 4

(b) If $\alpha=(1,\, 2,\, 0)$, $\beta=(3,\, -1,\, 1)$, and $\gamma=(4,\, 1,\, 1)$, determine whether they are linearly dependent or not. 4

3. (a) If $A=\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ $B=\begin{bmatrix} p & q & r \\ s & t & u \end{bmatrix}$, $C=\begin{bmatrix} l & m \\ n & k \\ h & g \end{bmatrix}$, then establish that $(A+B)C=AC+BC$. 4

(b) If $P=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, and $Q=\begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix}$ then establish 4

(i) $(P+Q)^T = P^T + Q^T$ and (ii) $(P.Q)^T = Q^T.P^T$.

4. (a) Prove that two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values of A are linearly independent. 4

(b) Prove that the eigen values of a real symmetric matrix are all real. 4

5. (a) Prove that a matrix is non-singular if and only if it can be expressed as the product of a finite number of elementary matrices. 4

(b) Prove that if the rank of a real symmetric matrix be 1 then the diagonal elements of the matrix cannot be all zero. 4

6. (a) Diagonaliza the matrix $A=\begin{bmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{bmatrix}$. 5

(b) Define a basis of a vector space. Do the vectors (1, 1, 2), (3, 5, 2) and (1, 0, 0) form a basis of $\mathbb{R}^3$? Justify. 3

7. (a) Find the eigen vectors and eigenvalues of $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ 4

(b) If Q_θ represents the matrix for rotation (in x-y plane) through an angle θ about the origin, prove that $Q_\theta^2 = Q_{2\theta}$ and $Q_\theta Q_{-\theta} = I_2$ 4

8. (a) State Cayley-Hamilton's Theorem and verify it for the matrix $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. 4

Hence find A^{-1}.

(b) What matrix has the effect of rotating every point through 90º and then projecting the result onto the x-axis? What matrix represents projection onto the x-axis followed by projection onto y-axis? 2+2

9. (a) Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$. 4

(b) If $A=\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Correct or justify: 4

(i) $(A-B)(A+B)=A^2-B^2$

(ii) $(A-C)(A+C)=A^2-C^2$

10.(a) Express $A=\begin{bmatrix} 2 & 5 & -3 \\ 7 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ as a sum of a symmetric and skew symmetric matrix. 3

(b) If $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $C=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ then verify that $AC=CA=6I_3$ and use this result to solve the system of equations 5

$x-y=3,\ 2x+3y+4z=17,\ y+2z=7$.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

——×——