WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 4th Semester Examination, 2020

MTMHGEC04T/MTMGCOR04T-MATHEMATICS (GE4/DSC4)

Time Allotted: 2 Hours

Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5=10$

(a) Prove that f is invertible when $f:\boldsymbol{R}\to\boldsymbol{R}$ be defined by $f(x)=3x+1,\ x\in\boldsymbol{R}$.

(b) Check whether the following relation ρ is an equivalence relation or not on the set of integers $\mathbb{Z}$. Justify your answer.

$$x\rho y \text{ if and only if } x-y \text{ is an even integer.}$$

(c) Examine that every cyclic group is abelien.

(d) In a group $(G,\,0)$, a is an element of order 30. Find the order of a^{18}.

(e) Find the order of the quotient group $\mathbb{Z}/10\mathbb{Z}$.

(f) Find the elements of the ring $\mathbb{Z}_{12}$ which are zero divisors.

(g) Show that the ring of matrices $\left\{\begin{pmatrix}2a & 0\\ 0 & 2b\end{pmatrix}: a,\,b\in\boldsymbol{Z}\right\}$ contains divisors of zero and does not contain the unity.

(h) Consider the ring $\mathbb{Z}$. In this ring, prove that $5\mathbb{Z}=\{5k:k\in\mathbb{Z}\}$ is an ideal of $\mathbb{Z}$.

2. (a) Let ρ be a reflexive and transitive relation on a set S. Prove that $\rho\cap\rho^{-1}$ is an equivalence relation on the set S. 3

(b) Let $f:\mathbb{R}^{+}\to\mathbb{R}^{+}$ and $g:\mathbb{R}^{+}\to\mathbb{R}^{+}$ be functions defined by $f(x)=\sqrt{x}$ and $g(x)=3x+1$ for all $x\in\mathbb{R}^{+}$, where $\mathbb{R}^{+}$ is the set of all positive real numbers. Find $f\circ g$ and $g\circ f$. Is $f\circ g=g\circ f$? 2+2+1

3. (a) Prove that inverse of a bijective function is also bijective. 4

(b) Let H be a normal subgroup of a group G. Prove that if G is commutative, then so is the quotient group G/H. 4

4. (a) If G is a group such that $(ab)^2 = a^2b^2$ for all $a, b, \in G$; then show that G must be abelian. 4

(b) Let H be a subgroup of a group G and $[G : H] = 2$. Then H is normal in G. 4

5. (a) Find an element $[b] \in \mathbb{Z}_9$ such that $[8]\cdot[b] = [1]$. Does $[b] \in U_9$? 3+2

(b) Let G be a group. Prove that $Z(G)$ is a normal subgroup of G. 3

6. (a) Let H be a subgroup of a group G. Then for all $a, b \in G$, either $aH = bH$ or $aH \cap bH = \emptyset$. 4

(b) If $\alpha = (1\ 2\ 5\ 7)$ and $\beta = (2\ 4\ 6) \in S_7$, find $\alpha \circ \beta \circ \alpha^{-1}$. 4

7. (a) Show that if p be a prime and a be a positive integer such that p is not a divisor of a then $a^{(p-1)} \equiv 1 \pmod{p}$. 4

(b) Prove that every group of order less than 6 is commutative. 4

8. (a) Show that all complex roots of $z^6 = 1$ form a group under the usual complex multiplication. 3

(b) Let R be the set of all even integers. Define addition as usual and multiplication by $a.b = \frac{1}{2}ab$. Show that R is a ring. 5

9. (a) Prove that the ring of matrices $\left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbf{R} \right\}$ is a field 4

(b) Prove that a field is an integral domain. 4

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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